MODELLING OF HEAT AND MASS TRANSFER AROUND SINGLE HOT PARTICLE SURROUNDED BY A VAPOR LAYER AND LIQUID*

A.A. Gubaidullin, I.N. Sannikov, and E.A. Kosheleva

Tyumen Department of Institute of Theoretical and Applied Mechanics SB RAS, 625000 Tyumen, Russia

Introduction

The investigation of heat and mass transfer process around a single hot particle surrounded by vapor layer and liquid is actual due to a study of vapor explosion in the time of hard accidents on the nuclear reactors and metallurgy industry. The process of heat and mass transfer of vapor bubble at surrounding liquid was investigated well [1]. However, insufficient attention was devoted a similarly process in the case of presence of a hot particle in the bubble. The models describing this process were proposed in the works [2, 3 and 4]. Unfortunately the questionable assumptions were used in the papers [2, 3] that can restricted the field of applications proposed ones. The model from the work [4] was developed just for the some cases.

The aim of this work is to fill up the mentioned gaps.

The basic equations

Consider the system of equations of model that describe the heat and mass transfer around hot particle surrounded by vapor layer and liquid. The equations of heat influx to a particle, vapor and liquid can be written as

$$\rho_d^\circ c_d \frac{\partial T_d}{\partial t} = \frac{\lambda_d}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_d}{\partial r}\right), \qquad r \in (0, d); \tag{1}$$

$$\rho_g^\circ c_{pg}\left(\frac{\partial T_g}{\partial t} + w_g \frac{\partial T_g}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_g r^2 \frac{\partial T_g}{\partial r}\right) + \frac{dp_g}{dt}, \qquad r \in (d, R(t)); \tag{2}$$

$$\rho_l^\circ c_l\left(\frac{\partial T_l}{\partial t} + w_l \frac{\partial T_l}{\partial r}\right) = \frac{\lambda_l}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_l}{\partial r}\right), \qquad r \in (R(t), \infty), \tag{3}$$

where ρ_i°, T_i $(i = d, g, l)$ are the densities and temperatures of particle, vapor and liquid; w_j, c_j, λ_j $(j = d, l)$ are the radial velocities, the specific heats and the heat conduction coefficients of particle and liquid; c_{pg} is the adiabatic specific heat of vapor; d and $R(t)$ are the radii of particle and vapor bubble accordingly. The liquid and particle consider incompressible. It is supposed that the pressure in vapor p_g is homogeneous in space and the dependence of thermal conductivity on temperature is linear:

$$\lambda_g = a_g T_g + \lambda_{g0}.$$

The vapor mass balance equation is

* The work was supported by the Russian Foundation for Basic Research (grant No. 01-01-00374) and Ministry of Education of Russian Federation (grant No. E00-4.0-77).

Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)
23 Aug 2002	N/A	-

Title and Subtitle Modelling of Heat and Mass Transfer Around Single Hot Particle Surrounded by A Vapor Layer and Liquid	**Contract Number**
	Grant Number
	Program Element Number
Author(s)	**Project Number**
	Task Number
	Work Unit Number
Performing Organization Name(s) and Address(es) Institute of Theoretical and Applied Mechanics Institutskaya 4/1 Novosibirsk 530090 Russia	**Performing Organization Report Number**
Sponsoring/Monitoring Agency Name(s) and Address(es) EOARD PSC 802 Box 14 FPO 09499-0014	**Sponsor/Monitor's Acronym(s)**
	Sponsor/Monitor's Report Number(s)

Distribution/Availability Statement
Approved for public release, distribution unlimited

Supplementary Notes
See also ADM001433, Conference held International Conference on Methods of Aerophysical Research (11th) Held in Novosibirsk, Russia on 1-7 Jul 2002

Abstract

Subject Terms

Report Classification unclassified	**Classification of this page** unclassified
Classification of Abstract unclassified	**Limitation of Abstract** UU

Number of Pages
5

$$\frac{d\rho_g^\circ}{dt}+\frac{\rho_g^\circ}{r^2}\frac{\partial r^2 w_g}{\partial r}=0\,,\qquad \frac{d}{dt}=\frac{\partial}{\partial t}+w_g\frac{\partial}{\partial r}\,. \tag{4}$$

In the case of incompressible liquid we have:

$$w_l r^2 = w_{lR} R^2\,, \tag{5}$$

where w_{lR} is the radial velocity of liquid near to a bubble surface.

We use the ideal gas state equation for vapor in a bubble:

$$p_g = c_{pg}\frac{\gamma-1}{\gamma}\rho_g^\circ T_g\,. \tag{6}$$

The Clapeyron – Clausius equation relates the pressure in a vapor and the saturation temperature T_s:

$$\frac{dT_s}{dp_g}=\frac{T_s}{\rho_{gs}^\circ \ell}\left(1-\frac{\rho_{gs}^\circ}{\rho_l^\circ}\right). \tag{7}$$

In this equations γ, ℓ and ρ_{gs}° are vapor adiabatic exponent, specific heat of vaporization and vapor density on the saturation line accordingly.

The Rayleigh – Lamb equation describes the oscillations of bubble:

$$R\frac{\partial w_{lR}}{\partial t}+\frac{3}{2}w_{lR}^2+\frac{4\nu_l}{R}w_{lR}=\frac{1}{\rho_l^\circ}\left(p_g-p_\infty-\frac{2\sigma}{R}\right),$$

$$\frac{dR}{dt}=w_{lR}+\frac{j}{\rho_l^\circ}=w_{gR}+\frac{j}{\rho_{gs}^\circ}\,.$$

where ν_l and σ are kinematic viscosity of liquid and surface tension coefficient, p_∞ is pressure in liquid far from bubble, j and w_{gR} are intensity of phase transition and velocity of vapor on boundary surface.

The system of equations (1) – (9) is complete. The initial and boundary conditions are required to set for posing a problem. The boundary conditions for the equations (1) – (3) are:

$$r=0:\qquad \frac{\partial T_d}{\partial r}=0\,; \tag{10}$$

$$r=d:\qquad \lambda_d\frac{\partial T_d}{\partial r}=\lambda_g\frac{\partial T_g}{\partial r}\,,\qquad T_d=T_g\,; \tag{11}$$

$$r=R(t):\qquad \lambda_l\frac{\partial T_l}{\partial r}-\lambda_g\frac{\partial T_g}{\partial r}=j\ell\,,\qquad T_g=T_l=T_s(p_g)\,; \tag{12}$$

$$r=\infty:\qquad \frac{\partial T_l}{\partial r}=0\,. \tag{13}$$

The boundary conditions (12) on bubble surface describe the phase transition liquid-vapor in a quasi-equilibrium approximation. Therefore first boundary condition in (12) is considered as the equation for calculation of intensity of phase change j. There is the following requirement for velocity of vapor w_g:

$$r=d:\qquad w_g=0\,. \tag{14}$$

It is supported that initial temperature in a particle, vapor and liquid is linear function.

Transformation of equations system

Similar to [1] we transform the system of equation (1) – (14) to a convenient form for numerical simulations. First of all we note that the equation (6) is inconvenient for a calculation of pressure in a vapor because the density ρ_g° and the temperature T_g is depended on spatial coordinate r. Besides we have not explicit equation for the calculation of vapor velocity w_g. The equations for w_g and p_g can be obtained by the substitute of expressed from equation (6) vapor density in the equation (4):

$$\frac{dT_g}{dt} = T_g\left(\frac{1}{p_g}\frac{dp_g}{dt} + \frac{1}{r^2}\frac{\partial r^2 w_g}{\partial r}\right). \tag{15}$$

Then the expression (15) is substituted in the equation of heat influx to the vapor (2). The retrieved expression is integrated on r from d up to $R(t)$. The result of these transformations is:

$$\frac{dp_g}{dt} = \frac{3}{R^3 - d^3}\left[(\gamma-1)\left(\lambda_g R^2 \left.\frac{\partial T_g}{\partial r}\right|_{r=R} - \lambda_g d^2 \left.\frac{\partial T_g}{\partial r}\right|_{r=d}\right) - \gamma p_g w_{gR} R^2\right]. \tag{16}$$

Repeat again the integration in limits from d and up to $r' < R(t)$ and account the condition (14) we obtain the expression for the radial velocity of vapor:

$$w_g(r) = \frac{1}{\gamma p_g r^2}\left[(\gamma-1)\left(\lambda_g r^2 \frac{\partial T_g}{\partial r} - \lambda_g d^2 \left.\frac{\partial T_g}{\partial r}\right|_{r=d}\right) - \frac{r^3 - d^3}{3}\frac{dp_g}{dt}\right]. \tag{17}$$

Here the prime over space coordinate r is neglected.

We use new system of coordinates where the surface of bubble is fixed by following transforming operators:

$$r \in (0,d): \quad \tau = t, \quad \eta = r; \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial r} = \frac{\partial}{\partial \eta}; \tag{18}$$

$$r \in (d,\infty): \quad \tau = t, \quad \xi = \frac{r-d}{R-d}; \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - \frac{W\xi}{R-d}\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial r} = \frac{1}{R-d}\frac{\partial}{\partial \xi}, \quad \frac{dR}{dt} \equiv W. \tag{19}$$

The system of equations (1) – (9) and equations (16) – (17) for w_g and p_g may be written in the new coordinates as

$$\frac{\partial T_d}{\partial t} = \frac{D_d}{\eta^2}\frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial T_d}{\partial \eta}\right), \qquad \eta \in (0,1); \tag{20}$$

$$\frac{\partial T_g}{\partial t} + \frac{w_g - W\xi}{R-d}\frac{\partial T_g}{\partial \xi} =$$

$$= \frac{D_g}{[(R-d)\xi + d]^2 (R-d)^2}\frac{\partial}{\partial \xi}\left([(R-d)\xi + d]^2 \frac{\partial T_g}{\partial \xi}\right) +$$

$$+\frac{a_g}{\rho_g^\circ c_{pg}(R-d)^2}\left(\frac{\partial T_g}{\partial \xi}\right)^2+\frac{1}{\rho_g^\circ c_{pg}}\frac{dp_g}{dt}, \quad \xi\in(0,1); \quad (21)$$

$$\frac{\partial T_l}{\partial t}+\frac{w_l-W\xi}{R-d}\frac{\partial T_l}{\partial \xi}=$$
$$=\frac{D_l}{[(R-d)\xi+d]^2(R-d)^2}\frac{\partial}{\partial \xi}\left([(R-d)\xi+d]^2\frac{\partial T_l}{\partial \xi}\right), \qquad \xi\in(1,\infty); \quad (22)$$

$$w_g(\xi)=\frac{1}{\gamma p_g[(R-d)\xi+d]^2}\left[\frac{\gamma-1}{R-d}\left(\lambda_g[(R-d)\xi+d]^2\frac{\partial T_g}{\partial \xi}-\lambda_g d^2\left.\frac{\partial T_g}{\partial \xi}\right|_{\xi=0}\right)-\right.$$
$$\left.-\frac{[(R-d)\xi+d]^3-d^3}{3}\frac{dp_g}{dt}\right], \quad (23)$$

$$w_l(\xi)=w_{lR}\frac{R^2}{[(R-d)\xi+d]^2}; \quad (24)$$

$$\frac{dp_g}{dt}=\frac{3}{R^3-d^3}\left[\frac{\gamma-1}{R-d}\left(\lambda_g R^2\left.\frac{\partial T_g}{\partial \xi}\right|_{\xi=1}-\lambda_g d^2\left.\frac{\partial T_g}{\partial \xi}\right|_{\xi=0}\right)-\gamma p_g w_{gR}R^2\right], \quad (25)$$

$$\frac{dT_s}{dt}=\frac{T_s}{\rho_{gs}^\circ \ell}\left(1-\frac{\rho_{gs}^\circ}{\rho_l^\circ}\right)\frac{dp_g}{dt}; \quad (26)$$

$$R\frac{\partial w_{lR}}{\partial t}+\frac{3}{2}w_{lR}^2+\frac{4\nu_l}{R}w_{lR}=\frac{1}{\rho_l^\circ}\left(p_g-p_\infty-\frac{2\sigma}{R}\right), \quad (27)$$

$$\frac{dR}{dt}=w_{lR}+\frac{j}{\rho_l^\circ}=w_{gR}+\frac{j}{\rho_{gs}^\circ}. \quad (28)$$

In these equations the time τ is renamed as t and the temperature conduction coefficients of particle, vapor and liquid is

$$D_d=\frac{\lambda_d}{\rho_d^\circ c_d}, \qquad D_g=\frac{\lambda_g}{\rho_g^\circ c_{pg}}=\frac{a_g}{\rho_g^\circ c_{pg}}T_g+\frac{\lambda_{g0}}{\rho_g^\circ c_{pg}}, \qquad D_l=\frac{\lambda_l}{\rho_l^\circ c_l}.$$

The boundary conditions (10) – (14) is transformed as

$$\eta=0: \qquad \frac{\partial T_d}{\partial \eta}=0; \quad (29)$$

$$\eta = 1 \ \ (\xi = 0): \qquad \lambda_d \frac{\partial T_d}{\partial \eta} = \frac{\lambda_g}{R-d} \frac{\partial T_g}{\partial \xi}, \qquad T_d = T_g, \qquad w_g = 0; \tag{30}$$

$$\xi = 1: \qquad \lambda_l \frac{\partial T_l}{\partial \xi} - \lambda_g \frac{\partial T_g}{\partial \xi} = j\ell(R-d), \qquad T_g = T_l = T_s(p_g); \tag{31}$$

$$\xi = \infty: \qquad \frac{\partial T_l}{\partial \xi} = 0. \tag{32}$$

We use second order approximation of derivatives on space for the system of equations (20) - (28) with boundary conditions (29) – (32). For integration on time we use the explicit difference scheme based on Runge – Kutta algorithm with the forth order of accuracy. This scheme was proposed in work [5] at the solution of problem of heat and mass transfer of vapor bubble in liquid.

Conclusion

The model is proposed that describes the heat and mass transfer around hot particle surrounded by a vapor layer and liquid. It was supposed that the dependence of thermal conductivity in a vapor on temperature is linear, the pressure in vapor is homogeneous in space. The liquid-vapor phase transition was taken into account in a quasi-equilibrium approximation. The liquid and the hot particle were considered incompressible.

The procedure of numerical integration of system of equations is developed that is based on the explicit difference scheme with the forth order of accuracy on time and second order accuracy on space.

REFERENCES

1. **Nigmatulin R.I.** Dynamics of Multiphase Media. N.Y.: Hemisphere Publ. Corp. 1991.
2. **Usynin G.B., Khramov N.I.** Vapor explosion in mixture of two liquids // Fizika goreniya i vzryva. 1983. № 3. C. 112 - 115. (in Russian)
3. **Zonenko S.I.** On numerical investigation of vapor envelope dynamics near hot solid particle in liquid // Izvestiya AN SSSR. Seriya MZhG. 1985. № 4. C. 154 – 157. (in Russian)
4. **Ganiev O.R.** Dynamics and heat and mass transfer of liquid with a “two-phase” bubbles: Ph.D. Thesis. Moscow, 1989. (in Russian)
5. **Nigmatulin R.I.** Fundamentals of mechanics of heterogeneous media. Moscow: Nauka, 1978. (in Russian)